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VIRTUAL EXPLORATION BY COMPUTING GLOBAL FAMILIES OF TRAJECTORIES WITH SUPERCOMPUTERS

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Previous research has focused on locating trajectories in the three-body problem useful for mission design using quasi-periodic orbits and their invariant manifolds. Although these techniques have been very successful, the question arises as to whether other useful trajectories exist which cannot be found using the current methodology. In this study, the advantages of a supercomputer are used to investigate a large number of trajectories for potential applications to mission design. The tools applied to this problem were the calculation of collision orbits and Poincaré maps. The collision orbit calculations were found to provide a guide to the types of orbits that led to Europa approach. Examining Poincaré maps over many energies revealed key aspects of the variation of invariant manifolds with these energy changes.

INTRODUCTION

The fact that no comprehensive set of solutions, such as has been found for the two-body problem, exists for the three-body problem makes finding trajectories for particular mission design applications difficult. Previous research has successfully located useful trajectories by utilizing known quasi-periodic orbits and their invariant manifolds[1, 2]. It is likely that other desirable trajectories exist that are associated with unknown quasi-periodic orbits, or in regimes that have simply not yet been analyzed. Therefore, it is desirable to search the design space in a systematic way while focusing on particular regions that may be useful to mission design. One simple way to do this in any model is by selecting many initial conditions in some region of interest and integrating the resulting trajectories using a supercomputer[3, 4]. This approach has an advantage in that it possesses the potential to be easily extended to N-body problems, or to models that include the true ephemerides of the bodies. Integrating trajectories for examination throughout the design space is currently infeasible even with supercomputers, so it is also desirable to use techniques that provide a theoretical basis for limiting the design space. Two such techniques which conveniently

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allow for the viewing of large numbers of trajectories in a relatively simple manner are the computation of collision orbits and Poincaré maps.

Impact trajectories or collision orbits are space trajectories which collide with a planet or moon. Understanding these trajectories is important for scientific, commercial, and planetary protection applications. The recent return of the Genesis spacecraft to Earth with its precious cargo of solar wind samples heralds a new epoch of sample return missions from deep space. The spectacular impact of comet Shoemaker-Levy 9 highlighted the potential danger of Near Earth Objects to Earth. At the same time, some impact orbits may be nudged into capture orbits so that the rogue asteroid or comet may be harnessed to provide nearly infinite resources to the Earth and the Moon. These in-situ resources will be crucial for the exploration and development of space. Other applications include not only planetary protection from Near Earth Object impacts, but also planetary quarantine from contamination of hazardous material and the problem of orbital debris. In all these applications, a fundamental understanding of the dynamics of impact orbits is crucial for solving the many problems they present.

Recent advances in nonlinear astrodynamics provide some new theoretical and computational approaches to the study of impact and capture orbits. We now know that much of the Solar System is connected by a network of low energy orbits forming tubes in space sometimes referred to as the "Interplanetary Superhighway" (see Lo[2]). The design of the Genesis Earth return trajectory was one of the space missions to use these tubes (see Lo et al.[5] and Howell et al.[1]). These tubes, known as invariant manifolds (high dimensional surfaces composed of trajectories in a fixed energy level) in dynamical systems theory, are generated by unstable periodic and quasi-periodic orbits. Poincaré first predicted these invariant manifolds in his celebrated study of the Three-Body Problem in the late nineteenth century[6]. In fact, it was through this study of invariant manifolds that he first discovered deterministic chaos where a small change in the initial conditions of a trajectory can greatly alter its subsequent path. This, in fact, is the mechanism controlling low energy orbits.

We know from numerical work and experience that impact orbits are not all related to invariant manifolds of libration orbits despite their importance. There are uncountable families of unstable orbits in the vicinity of a planet or moon, suggesting that a large scale numerical exploration of impact orbits at different energy levels would be useful. The data can be interpreted by our knowledge of invariant manifold theory. In this way, we may quickly gain insight into the key families of unstable orbits and their manifolds that control the impact trajectories. This problem is obviously well suited for parallel computing given the fact that there are so many individual trajectories to examine. The deceptive simplicity of the problem belies the complex dynamics and organization of these orbital families. Moreover, this particular problem is but the starting point for the significantly more ambitious project of mapping the Interplanetary Superhighway.

An additional approach that is suitable for implementation on a parallel supercomputer is the computation of Poincaré maps. Although these computations are primarily restricted to the Circular Restricted Three-Body Problem (CRTBP), the results may be applied to models including the full ephemeris by using differential correction. In their simplest form, these Poincaré maps reduce the dimension of a system and record information about large numbers of trajectories as they pass through a plane. Various methods of plotting Poincaré maps allow key characteristics of these trajectories, along with their relationship to known dynamical structures such as quasi-periodic orbits, to be quickly examined. One area where these techniques are currently of particular importance is for satellite tours in the Jovian system[7, 8, 9].

These uses of a theoretical understanding of the dynamics to direct the computation and interpret the results should provide a more intelligent approach to the computation. This theoretical basis should reduce the required effort compared to the brute force technique and increase our insight into the problem. The specific problem that is currently of interest and which is chosen as an example in this study is the proposed low thrust JIMO mission. Two of the primary regimes that are of interest in the trajectory design for this mission are the Europa approach and resonance transitions as the spacecraft navigates between the moons[8]. These two regimes can be directly related to the calculation of collision orbits and Poincaré maps, respectively. Furthermore, calculation of these trajectories over different energies gives an idea of how the design space varies with low thrust. The results presented in this study are part of the general development of numerical tools for low thrust trajectory design using dynamical systems techniques.

The Circular Restricted Three-Body Problem

The CRTBP was the primary model used in this analysis. In this problem two massive bodies exist with the larger being referred to as the primary and the smaller as the secondary. These two bodies are assumed to rotate about their center of mass in circular orbits, and the objective is to describe the motion of a third infinitesimal mass placed in this system. If the infinitesimal mass is restricted to the plane of motion of the two primaries, the problem is called the planar or coplanar CRTBP (PCRTBP). In formulating the equations of motion for the infinitesimal mass, the required quantities are usually made dimensionless so that the mass of the secondary is μ, and the primary has mass $1 - \mu$. The distance (d) between the two massive bodies becomes unity with the primary located on a rotating x-axis at $x = -\mu d = -\mu$ and the secondary at $x = (1 - \mu)d = 1 - \mu$. The dimensionless time corresponds to the angle between the x-axis of the rotating frame (defined so that the x-axis always passes through the two primaries) and the x-axis of the inertial frame. The period of the rotating system becomes 2π. Both the mean motion and the gravitational constant are unity. Using this notation, the equations of motion for the infinitesimal mass in the rotating system may be written as

$$
\begin{aligned}
\ddot{x} - 2\dot{y} &= x - (1-\mu)\frac{x - x_1}{{r_1}^3} - \mu\frac{x - x_2}{{r_2}^3} \\
\ddot{y} + 2\dot{x} &= \left(1 - \frac{(1-\mu)}{{r_1}^3} - \frac{\mu}{{r_2}^3}\right) y \\
\ddot{z} &= -\left(\frac{(1-\mu)}{{r_1}^3} + \frac{\mu}{{r_2}^3}\right) z.
\end{aligned}
\tag{1}
$$

Here the distances from the infinitesimal mass to the primary and secondary are r_1 and r_2, respectively. The x-axis location of the primary is x_1, and that of the secondary is x_2. The value of μ used for the Jupiter-Europa system in this study was approximately $\mu = 2.526645 \times 10^{-5}$. An integral of motion exists in this model called the Jacobi Constant which may be written as

$$\mathrm{C} = 2\Omega - \mathrm{V}^2 = x^2 + y^2 + \frac{2(1-\mu)}{r_1} + \frac{2\mu}{r_2} - \dot{x}^2 - \dot{y}^2 - \dot{z}^2. \tag{2}$$

Here, $\dot{x}$, $\dot{y}$, and $\dot{z}$ are the time derivatives of the position and V is the velocity in the rotating frame. Ω is the effective or augmented potential. Note the constraint that $V^2 > 0$ limits the resulting motion for a fixed Jacobi Constant. Five equilibrium points exist in this problem with the three colinear equilibrium points found along the line intersecting the primary and secondary. L_1 is defined here to be the equilibrium point between the primary and secondary. L_2 is on the opposite side of the secondary from L_1, and L_3 is on the opposite side of the primary. See Roy[10] or Szebehely[11] for detailed descriptions of the CRTBP.

COLLISION ORBITS

Theory & Application to Parallel Computing

The idea for the first avenue of analysis has its origins in the study of collisions, which have been examined extensively in the mathematical literature[12, 13, 14]. As would be expected, a double collision is said to occur when the position of two bodies in the system are the same and a singularity in the equations of motion occurs. This double collision is a particularly interesting case in astrodynamics, as this event would roughly correspond to the case of a spacecraft encountering a planet or moon, which is often the focus of a mission. Knowledge of the trajectories that encounter the singularity, or the collision orbits, could be very useful. Various methods of regularization have been developed to account for this singularity, but parts of the development of a particular method by Easton[12] give a theoretical basis for the technique used to compute collision orbits in this analysis. To briefly summarize, an isolating block can be constructed that contains the singularity and which removes it from the manifold where the vector field is defined. In a simple case, this could correspond to removing a disk from around a singularity in a planar problem. The regularization then requires that points on the boundary of the disk be paired with each other using the vector field. For this study, the problem reduces to that of selecting a small disk or sphere around the body of interest. The collision orbits in the limit as the disk or sphere becomes smaller are those traveling toward the center of mass of the body with a velocity perpendicular to the edge of the disk or the surface of the sphere. For the case of Europa, the small size of the moon in relation to the total system makes it reasonable to choose the radius of the sphere to be on the order of the radius of Europa. This does in fact appear to be an acceptable size as a short study showed negligible changes in the qualitative nature of the results as the radius was increased or decreased by several factors. The velocities may then be constrained by selecting a specific Jacobi constant for the trajectories.

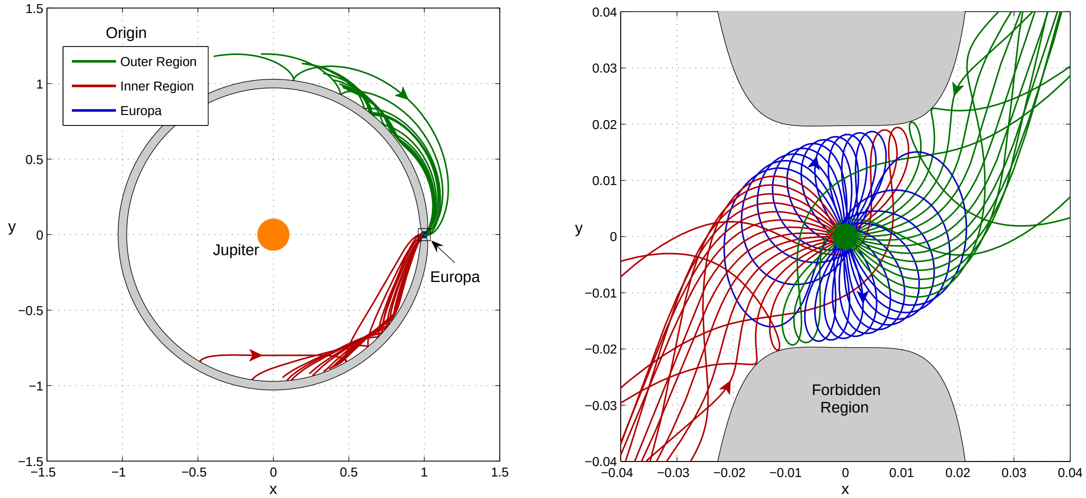
The actual computation of the collision orbits is then very straightforward. A sphere is constructed around the singularity (Europa's center of mass) in position space. Points on the sphere are then selected at even intervals with the velocity directed inward with a magnitude given by the Jacobi constant. A backward integration is then used to determine the point of origin of each collision orbit over a desired time interval. While this analysis can provide many interesting and potentially useful trajectories, it must be realized that most mission design applications would require the spacecraft to approach some point near the surface of Europa tangentially for orbit insertion or at a shallow angle for a lander mission. At least for the case of escape trajectories, Villac and Scheeres[15] showed that the minimum ΔV for escape from circular orbits was tangential. Many of the collision orbits could possibly be modified slightly to achieve these ends. Trajectories with velocities tangential to the surface could also simply be computed directly. In order to gain some understanding of these trajectories, this was done for cases with trajectories coming from the North, South, East and West, with North defined perpendicular to the plane of Europa's motion about Jupiter. Once the final conditions are integrated backward in time, they are categorized based on the point of origin for the integration time interval. The current categories are those trajectories that originated at Europa, those that come from the region inside the orbit of Europa, those that come from the region outside the orbit of Europa, and those that did not originate at Europa over the time interval of interest but have remained within 20 radii of the moon. Orbits in this last region may exist as temporarily captured orbits, a topic explored by Paskowitz and Scheeres[16]. The focus here is primarily on approach trajectories coming from the inner or outer regions. It should also be noted that although this analysis involves collision orbits integrated backward in time, symmetries in the PCRTBP allow the transformations $y \to -y$ and $t \to -t$. Using these, this analysis could be extended to trajectories escaping Europa.

For the spatial results in this paper, the number of points around the equator (defined to be in the orbital plane of Europa around Jupiter) was selected to be 180. Normally, 83 latitudes were chosen with the number of points at each latitude a function of the cosine of the latitude. Time intervals varied from π to 3π dimensionless time units. The computation of the trajectories was typically divided evenly among 32 processors of the parallel supercomputer. The primary supercomputer used in this study was the JPL SGI Altix 3000. It possesses 64 Intel Itanium 2 processors running at 900 MHz.

Comparison of Models

As the CRTBP has been widely studied and is well understood, it would be desirable to perform the analysis using this model. The question arises though as to whether the results obtained using this model will be applicable in the more general problem with multiple bodies using the true ephemerides of the moons. The CRTBP has usually been found to be sufficiently accurate for most mission design problems, however a limited comparison was performed to verify that this holds true in the case of collision orbits.

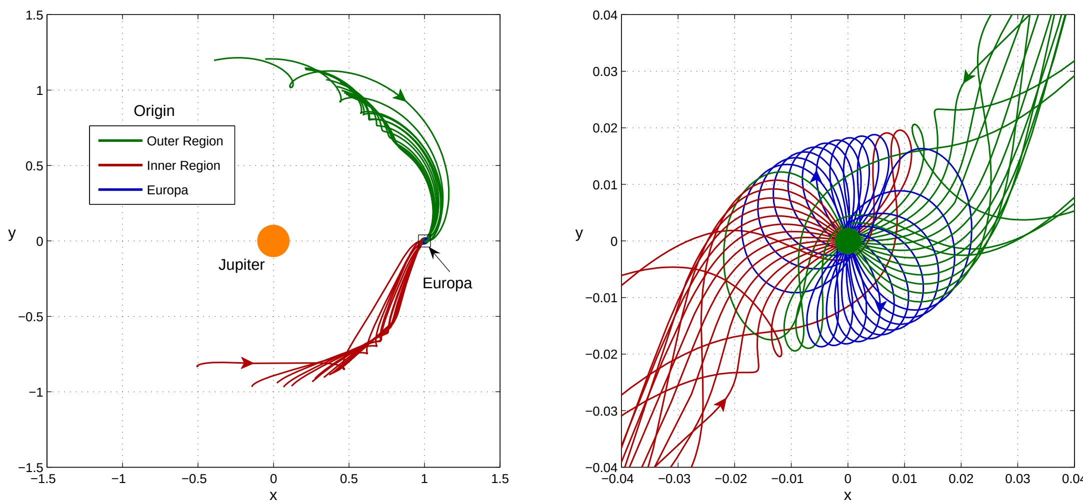
For this study, 50 trajectories were chosen that terminated on the surface of Europa and which traveled in the plane of Europa's orbit about Jupiter. They were given a velocity perpendicular to the disk representing Europa, and the magnitude of the velocity was



(a) Overview of trajectories

(b) Boxed region centered on Europa

Figure 1 Planar collision orbits integrated in the CRTBP for two revolutions of the secondary around the primary.



(a) Overview of trajectories

(b) Boxed region centered on Europa

Figure 2 Planar collision orbits integrated including the ephemerides of Jupiter's four primary moons for two revolutions of the secondary around the primary. Note that the forbidden region is not shown here since this is not in the CRTBP.

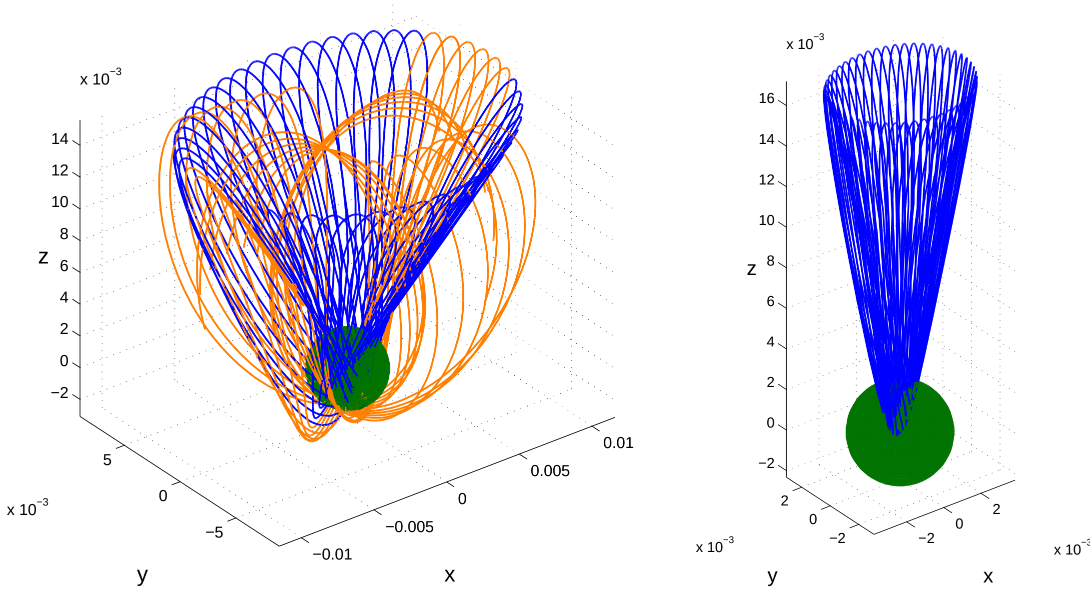
computed so that they all possessed the same Jacobi constant ($C \approx 3.0025$). This Jacobi constant was selected based on previous experience with Europa approach trajectories[8]. Of course the Jacobi constant does not apply in the case with ephemerides, so it was approximated using the CRTBP quantities for this case. Each trajectory is then integrated backward for two periods of the secondary around the primary to determine their point of origin over this time. This procedure was performed both in the CRTBP and in a case including the JPL ephemerides of Jupiter and its four primary moons.

Figure 1 shows the resulting trajectories in the PCRTBP. It can be seen from this figure that there are several obvious ways to categorize the various collision orbits. In this figure, they are categorized by their point of origin, and it can already be seen that distinct regions on Europa are directly related to the point of origin. Trajectories on the sides nearest the forbidden region generally originate at Europa and then impact the surface. Trajectories impacting the side toward Jupiter usually originate in the inner region, and trajectories traveling from the outer region usually impact on that side of Europa. It is interesting though, that several small regions on Europa do not follow this trend, which is a piece of information that could be useful for mission design. The results for the trajectories computed using the ephemerides are shown in Figure 2. The trajectories near Europa are not noticeably different in the two models although one trajectory that impacts Europa in the CRTBP was found to barely miss it in the ephemeris case. Only small differences are observed as the spacecraft travels further from Europa. The two cases generally appear to be topologically similar over the given time period, and as the focus of this study is on the qualitative nature of the results, the similarities of the collision orbits in the two models are sufficient to make the use of the CRTBP practical. Future studies may examine longer time periods in which case this conclusion would need to be reevaluated.

Spatial Collision Orbit Results

A spatial study of collision orbits revealed several interesting features. In general, trajectories impacting Europa at the lower latitudes had characteristics similar to those found in the planar study. At the higher latitudes, however, the impact trajectories tended to either originate at Europa, or remain near Europa for long periods of time. Figure 3 shows two cases with trajectories impacting at specified latitudes in order to illustrate this point. The 60^o latitude case shows that most of the collision orbits come upward with a cone-like structure. Remember that the integration is moving backward in time. Most then impact Europa, but some miss it and succeed in avoiding impact over the given time period. The 80^o latitude case more clearly shows the structure of the impact orbits originating at Europa, and for this energy none of the orbits originate elsewhere.

The same procedure can be performed for varying latitudes in order to examine the characteristics of the collision orbits, but it quickly becomes obvious that the large number of plots and trajectories make it difficult to apply the results to mission design objectives. When the added dimension of changing energy is included in the problem, the need for a different method of viewing or analyzing the data is clear. One method for viewing the results is to plot each trajectory as a point on the surface of the moon with a color corresponding to some metric. As mentioned in the planar study, one metric that can be used is some classification of the point of origin of the trajectory. The results for the case with the velocity normal to the surface of Europa are given in Figure 4. The most efficient method for examining the data has been found to be the generation of movies with each frame representing a particular Jacobi constant. The frames shown in this figure represent just a sample of the results that are typically generated. The range of Jacobi

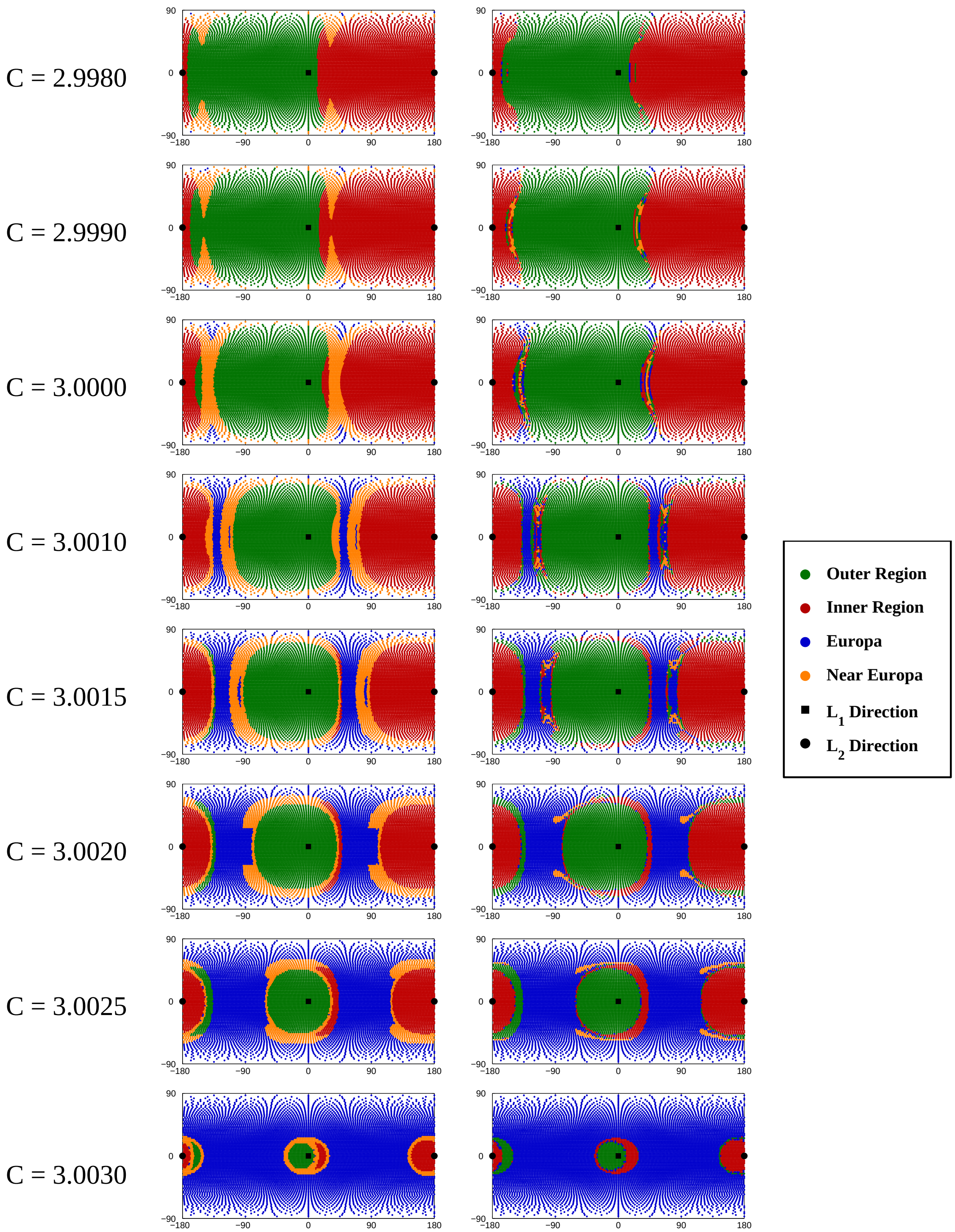


(a) 60^o Latitude (b) 80^o Latitude

Figure 3 Collision orbits impacting Europa at different latitudes. The blue trajectories both originated at Europa and impacted Europa. The orange trajectories did not encounter Europa over the dimensionless time interval of π.

constants selected for this analysis was based on previous experience with Europa approach trajectories. Examining the frames in Figure 4(a) it can be seen that at the lowest Jacobi constant nearly half the trajectories travel into the inner region and half to the outer region. Only a very few remain near Europa. As the Jacobi constant increases (or the trajectories become less energetic) more cases remain near Europa, and a higher percentage of the cases originate at Europa, with most of these cases falling on the dividing line between the outer and inner region trajectories. The results are perhaps most interesting at C = 3.0025 where it can be seen that near areas where most trajectories travel toward the outer (inner) region there are a very few adjacent to them that travel to the other region. Finally, with the highest Jacobi constant, the majority of cases that impact Europa originate at Europa. Although it is not shown here, for greater Jacobi constants all cases originate from Europa. This makes sense as the Jacobi constant is approaching the values for L_1 and L_2 which are approximately 3.003641 and 3.003608 respectively.

The next question that arises is as to how the results may vary with the integration time intervals of the collision orbits. Comparing the frames in Figures 4(a) and 4(b) shows that at first glance the general trends of the π and 3π interval cases are the same. However, it is a very interesting occurence especially at the lower Jacobi constants that after 3π time units some of the trajectories found on the borders of the various categorizations appear to move from one classification to another. Especially at C = 2.9990, it can be seen that some trajectories that at π were in the outer region had fallen back to the inner region at 3π and vice versa. The division between the categories now appears to contain trajectories of all types. This indicates that trajectories in these areas are especially chaotic as small



(a) π Integration (b) 3π Integration

Figure 4 Normal collision orbit case. The horizontal axes are longitude in degrees and the vertical axes are latitude in degrees.

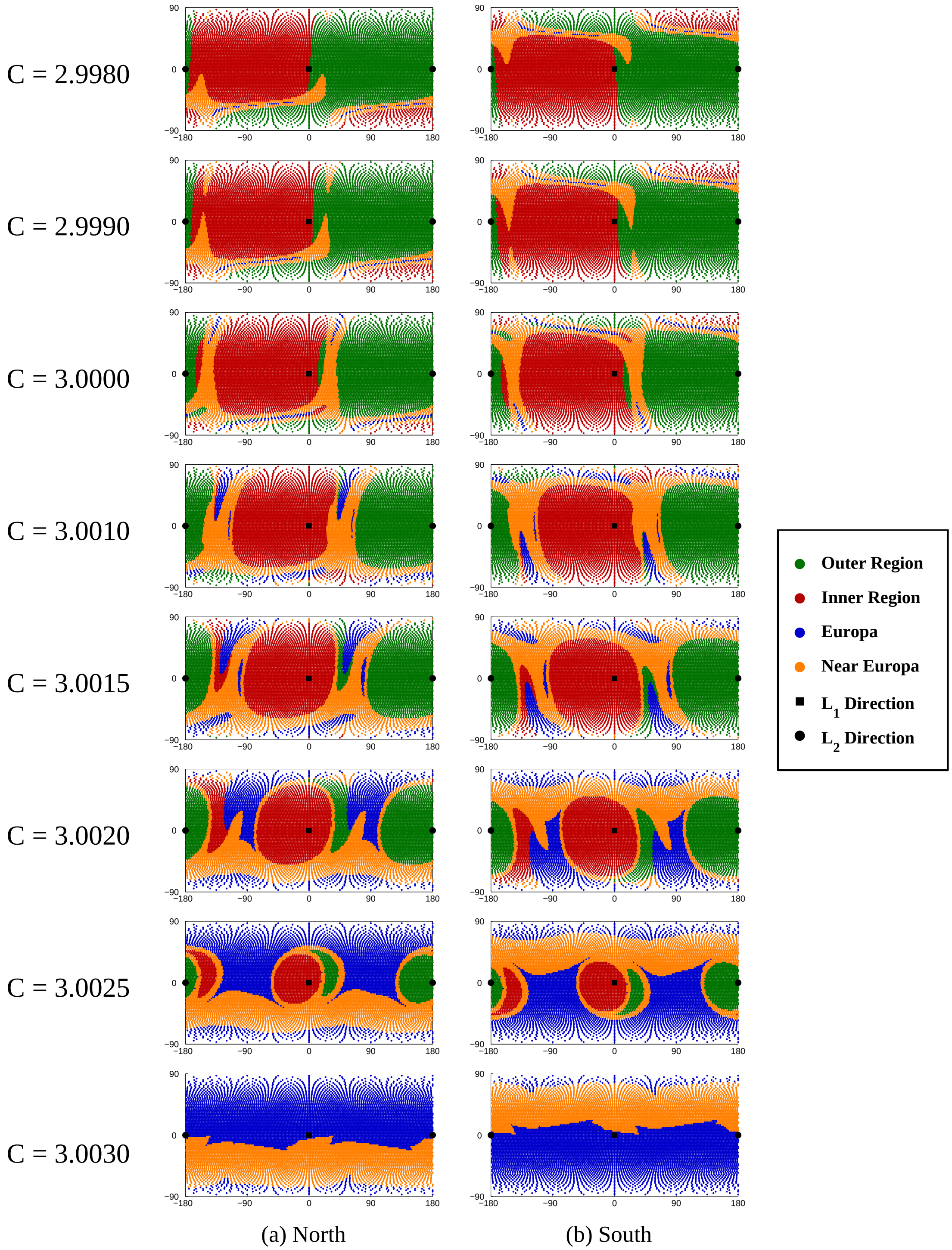


Figure 5 Tangential orbit case integrated for π time units. The horizontal axes are longitude in degrees and the vertical axes are latitude in degrees. The labels indicate the direction the trajectory was traveling from when it encounters Europa's surface.

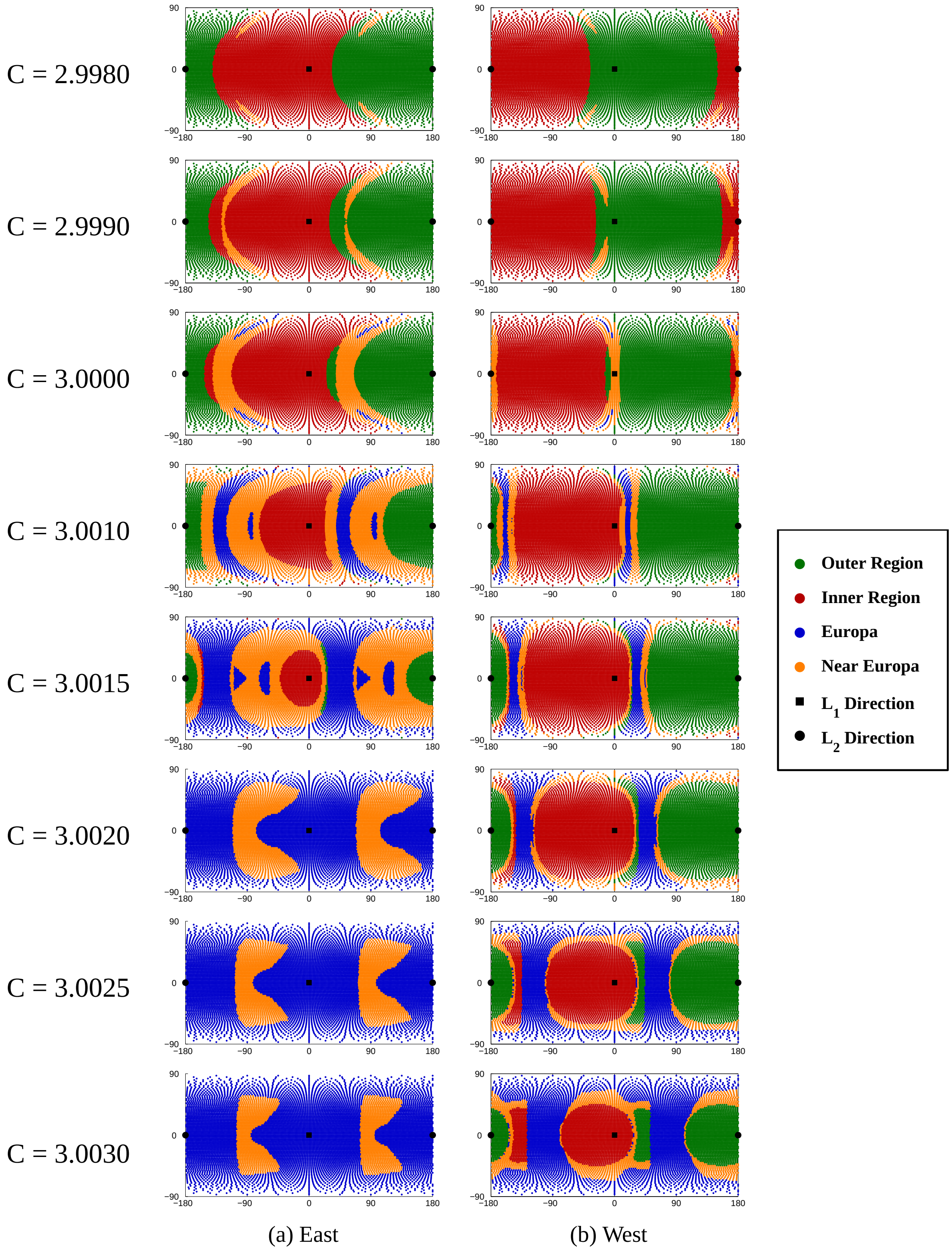
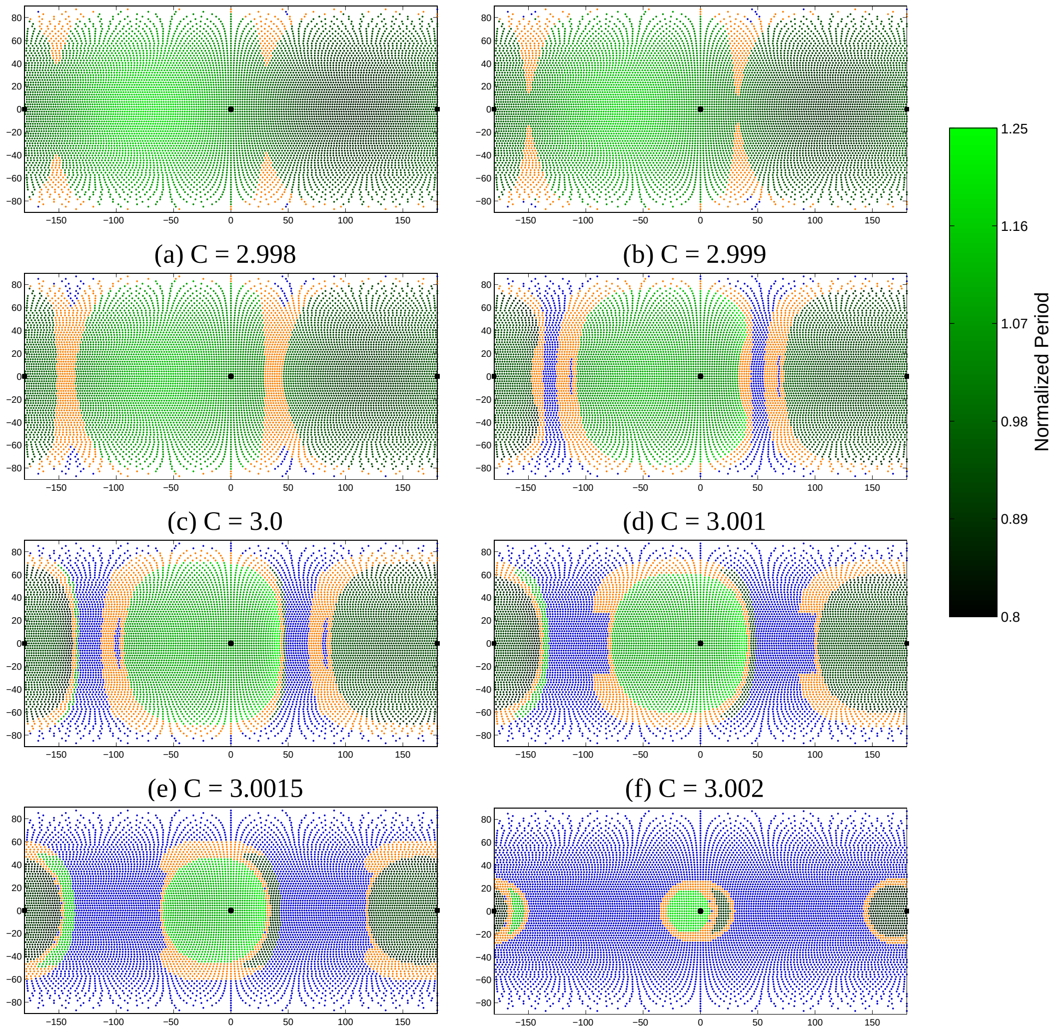


Figure 6 Tangential orbit case integrated for π time units. The horizontal axes are longitude in degrees and the vertical axes are latitude in degrees. The labels indicate the direction the trajectory was traveling from when it encounters Europa's surface.



(a) C = 2.998

(b) C = 2.999

(c) C = 3.0

(d) C = 3.001

(e) C = 3.0015

(f) C = 3.002

(g) C = 3.0025

(h) C = 3.003

Figure 7 Normal collision orbit case with shading corresponding to the period (normalized by Europa's Period) for trajectories originating in the inner or outer regions. The horizontal axes are longitude in degrees and the vertical axes are latitude in degrees.

changes in the impact conditions have resulted in large changes in their point of origin. It is these types of trajectories that have the potential to aid in the discovery of new routes among the moons and planets. Future work will focus more on longer integration times as newer computers become available, but the longest time interval that is feasible to run within a reasonable time period on the supercomputer is currently π time units. Larger time intervals require dividing the run up over several days or weeks.

The two additional cases in Figure 5 were run for trajectories approaching Europa's surface tangentially from the North and the South with integration time intervals of one π. It is immediately obvious that a symmetry exists between the two cases. As before, there appear to be two distinct areas that have trajectories originating in the inner and outer regions and as the Jacobi constant increases most trajectories originate at Europa. The difference seems to be that in the Northern case, the Southern trajectories tend to remain closer to Europa, and the opposite is true for the Southern case.

Figure 6 shows the results for the East and West tangential cases. Here obvious differences between the two cases can be observed. At C = 2.9980 they do appear quite similar, with trends similar to those seen in the earlier cases. However, with increasing Jacobi constant more trajectories remain closer to Europa for the East case, and by C = 3.0020 all trajectories either impact Europa or are within 20 radii of it. On the other hand, the West case has even more trajectories still in the outer or inner region than the normal case. Once again it is worth noting, that trajectories near the area separating the regions have the potential to travel to vastly different areas. This could be of value in targeting locations for orbit insertion given knowledge of whether the spacecraft is coming from the outer or inner region.

While these general classifications have the potential to serve as a useful road map for the design of approach trajectories, more specific information will typically be required for a given mission. Given the state at the origin of the particular trajectory, many metrics may be easily calculated. The two-body period is an example of such a metric that is currently of interest for low thrust trajectory design in the proposed JIMO mission. The two-body period calculated with respect to the barycenter and normalized by Europa's period for the inner and outer region trajectories is shown in Figure 7. This normalized period gives an overall indication of the resonances that are associated with collision orbits at Europa. This can in turn give information on the number of flybys needed before the Europa approach. In combination with tangential trajectory plots, this information can be very helpful as it also gives a range of resonances that might be obtained for a given energy in a time interval of π, and it helps provide information on the energy that might be necessary to approach Europa at a given resonance. By examining the periods of these normal collision orbits over the selected range of energies, it was found that it was possible to reach normalized periods ranging from approximately 0.83 to 1.24. This upper value corresponds roughly to the observation that a 5:6 resonant orbit (Europa period:spacecraft period) is typically required for Europa approach, although that was for a specific energy[8]. The maximum and minimum values tended to occur more closely to C = 3.0015 than any other value. However, the trends were generally not linear as can be seen from Figure 7. This might arise in part from the fact that many orbits that originate at a particular resonance at one energy might originate at Europa at a slightly different energy. Overall it appears that, analyzing particular trajectories from these maps should help guide research efforts to find useful trajectories that would not otherwise have been as quickly discovered utilizing the current research strategy.

POINCARÉ MAPS

Theory & Practical Computation on a Supercomputer

Poincaré maps are used in order to obtain a simplified discrete system from a more complicated continuous flow. This is done by reducing the dimension of the system, which at first might appear to result in a loss of information. However, it has been found that Poincaré maps are typically very useful for bringing out certain types of information that would otherwise be obscured. In order to compute a Poincaré map for a system in $\mathbb{R}^n$, a 'hypersurface' or surface of section in $\mathbb{R}^{n-1}$ is placed transverse to the flow. A trajectory intersecting the surface of section is integrated until it intersects the surface of section once again. The mapping is from the first intersection to the next intersection and so on. In this study, the points of the mapping are plotted using x and $\dot{x}$. These Poincaré surface of sections or Poincaré sections bring out a surprising amount of detail of which the location of stable periodic and quasi-periodic orbits are the most obvious features. For the planar CRTBP in $\mathbb{R}^4$, the surface of section is specified by fixing one of the coordinates, typically so that $y = 0$ in order to produce a surface in $\mathbb{R}^3$.

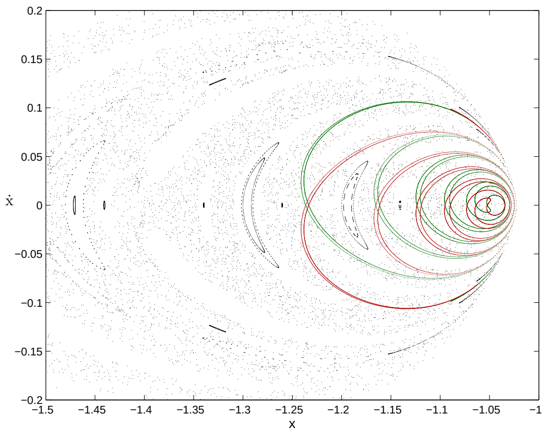
The Poincaré sections here were computed by starting with a set of points on the x-axis with a velocity in the y direction and integrating them forward in time. This integration was performed using a Runge-Kutta Fehlberg seventh-order integrator with stepsize control[17]. After an initial startup time, the intersections of the trajectories with the surface of section were recorded and plotted in the Poincaré section. Remember that with x defined, $\dot{x} = 0$, and $y = 0$, the magnitude of $\dot{y}$ can be calculated in the planar problem from the Jacobi Constant in Equation 2.

Although the computation of Poincaré sections over various energies, is in itself an interesting study, the focus of this paper is also related to how trajectories in Poincaré sections interact with known families of orbits and dynamical structures. Specifically, the problem chosen here is to examine how the invariant manifolds of Lyapunov orbits about L_2 in the planar problem change in relation to the structures in the Poincaré sections as the Jacobi constant varies. The Lyapunov orbits of interest were computed using a modified single-shooting method based on the symmetry of these orbits about the x-axis. Once an initial orbit had been found, the remaining orbits at different energies were easily computed by varying the value of the x-axis crossing. A secant method was used to interpolate initial conditions to produce orbits of any selected Jacobi constant within the original range. Once these orbits were obtained, both the stable and unstable invariant manifolds could be calculated. Their intersections with the desired Poincaré sections were then found and plotted over the desired energy range.

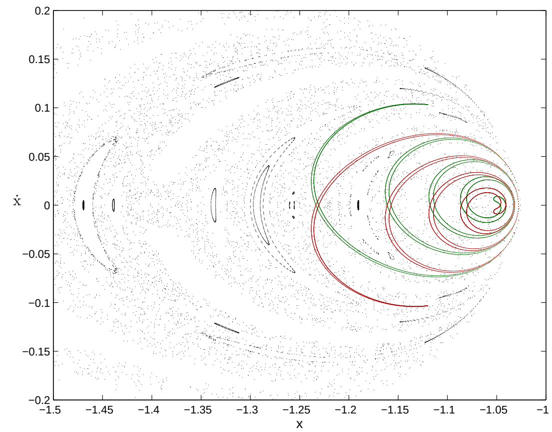
Implementing these algorithms on the parallel supercomputer was relatively straightforward. Once the initial conditions of all the desired trajectories for the background points were computed they were then divided up evenly and sent to each of 32 processors. The computation of the manifolds was divided among processors according to energy levels.

Effect of Varying the Jacobi Constant

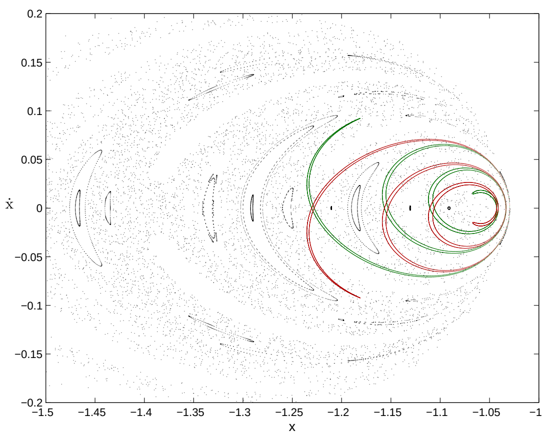
The Poincaré sections plotted in Figure 8 are a sample of the results generated to study the interaction of the invariant manifolds of Lyapunov orbits with the quasi-periodic orbits in the Poincaré sections. The distorted elliptical shapes, or islands, in the Poincaré sections represent stable quasi-periodic orbits with a stable periodic orbit at their center. Unstable periodic resonant orbits may be found in the chaotic sea between the islands, and some have been found to reside within the invariant manifolds of the Lyapunov orbits[8]. A



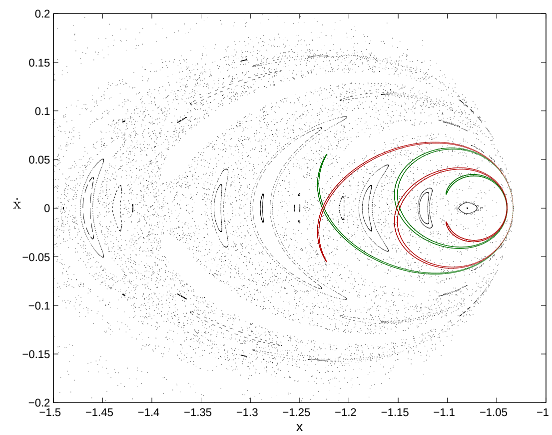
(a) C = 3.0015



(b) C = 3.0020



(c) C = 3.0025



(d) C = 3.0030

Figure 8 Representative Poincaré sections at different energy levels. The red (green) points are the unstable (stable) manifold.

general view of the evolution of these structures with energy may be obtained by comparing the background points of each Poincaré section. In general the islands at each resonance appear to move toward the right in these plots with changing shapes. It is difficult to compare the shapes though, as this is a function of the grid used to generate the Poincaré section in addition to the energy.

What is most interesting here is related to the changes in the invariant manifolds of the Lyapunov orbit with energy. At C = 3.0015, the invariant manifolds tend to wrap around many times with a large number of intersections. For at least some energies, these intersections contain unstable resonant orbits, and this relationship makes it possible for orbits near these resonances to approach Europa. So the changing nature of these intersections could have important consequences for Europa approach. It appears that if some of the intersections at key resonances disappear, this could provide a limit for the energy needed for Europa approach. This is an area that is currently being researched.

ROAD MAP FOR MAPPING THE INTERPLANETARY SUPERHIGHWAY

One of the trademarks of the new paradigm for mission design is the need for computing large families of trajectories. For the N-Body Problem, although theory tells us that invariant manifolds can provide parameters of the design space, they must now be computed by integrating differential equations. Once a map of the design space is obtained, designers can pick and choose multiple pathways and evaluate their performance just as they have done with orbital elements in the past. But, the computation of such maps is a substantial challenge. Consider the fact that even for the class of libration orbits around L_1 and L_2, this has not been done.

The use of dynamical systems theory for trajectory design was initiated by Carles Simó his group in Barcelona in the mid 1980's. This is truly a paradigm shift that has already and will continue to impact the aerospace community. One of the reasons for its power, much of which is still untapped, is that it provides structures in phase space which produce near optimal transfers in the N-Body Problem. This provides the N-Body analogy to the Two-Body tools such as Hohmann Transfers and Lambert's Theorem. Another problem in the N-Body regime is the lack of good orbital elements. Without orbital elements, there is no way to know what families of orbits are available for mission design. Invariant manifolds and the periodic orbits which generate them solve this problem. They tell us how to systematically map out the available orbital paths in space. However, this mapping must be done semi-analytically where large computations are required. This is where the need for massively parallel computation is called for.

Low energy orbits use the nonlinear dynamics of the N-Body Problem to provide transfers in space that would cost more using conic arcs provided by Lambert's solutions. In some instances, this cost difference can be significant. Moreover, low energy orbits provide orbits in space which are not obtained from conic approximations. Conley[18] first coined the term "Low Energy Orbits" in his study of transfers to the Moon using the nonlinear dynamics of the Three-Body Problem in the 1960's. However, he rejected them since they require much longer flight time than was considered acceptable at the time.

The purpose of this paper has been to introduce this set of problems and their relations to one another from a new perspective. This begins with simple questions such as "How do rocks from Mars or the Moon get to Earth?" The answer to this question is extremely complex, but recent advances in the understanding of the dynamics of transport within the Solar System provide significant insights into this problem[19, 20]. The results of this paper have given a hint of the discoveries that might be made using a combination of massive computations on supercomputers with dynamical systems theory. Further combinations of techniques that have not yet been considered have the potential to guide even new avenues of research.

CONCLUDING REMARKS

Searching the design space of approach orbits at Europa using collision orbits and orbits tangential to Europa's surface in a global search, indicated that trajectories in drastically different areas can ultimately lead to Europa. Various metrics can be used to provide information about these orbits, and as an example, the calculation of period was used to show that it might be used as an initial estimate in bounding the resonances that lead to Europa approach. Examination of the trajectories over different energies indicated where approach trajectories of different types might be possible, and the generated maps could be used as initial guesses in targeting these trajectories. Analyzing the maps showed that trajectories very close to each other at Europa could lead to very different regions, especially over long time periods as was hinted at in the longer integrations.

The examination of Poincaré maps over different energy levels showed that the structures related to quasi-periodic orbits in Poincaré sections vary noticeably with relatively subtle changes in energy. Significant changes were also observed in the invariant manifolds of the Lyapunov orbits along with a reduction in the number of intersections observed as the Jacobi constant increased. This indicates that as energy changes the effect on the invariant manifolds of these Lyapunov orbits cannot be neglected.

FUTURE WORK

Future work will include an examination of other possible metrics that have potential uses in mission design. The results of the 3π case indicate that it would be interesting to examine longer time periods. These longer time periods allow the effects of chaos to be more clearly observed as very different conditions may be reached with small changes in initial, or final, conditions. In the case of the Poincaré sections, the relationship between resonant orbits and the invariant manifolds of Lyapunov orbits is currently being examined. The effect of even greater energy variations will soon be incorporated. Ultimately, it would be desirable to use this as a tool to provide limits useful in mission design. Finally, it would be relatively easy to apply these techniques to other systems such as the Earth-Moon system or the Sun-Earth system, and some preliminary work has been performed in this area.

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