Astrodynamics Applications of Dynamical Systems Theory

ASEN6519 - Fall 2007
Tuesday/Thursday 12:30 – 1:45 pm
ECCR 1B08

Overview:

The course will focus on applying dynamical systems tools to astrodynamics primarily within the context of the three-body problem. Some of the areas of concentration will include the computation of periodic orbits, differential correction, invariant manifolds, and stability techniques. The homework will emphasize the development of algorithms that incorporate these methods while learning about their mathematical basis. Research projects will be assigned in order to include current topics of interest. Grades will be based primarily on the homework and research projects.

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Grading:

Homework:	60%
Mid-term Project:	15%
Final Project:	25%

Outline:

1. Dynamical Systems Overview, Terminology, Sample Applications
    a. Nonlinearity
    b. Phase space trajectories
    c. Autonomous and non-autonomous systems
    d. Topology (see Marsden)
2. Flows on the line
    a. Fixed points and stability
    b. Thinking geometrically or qualitatively (Poincaré)

    c. Stability analysis
    d. Existence and uniqueness
    e. Potential
    f. Lagrangian, Hamiltonian (see Landau and Lifshitz, Arnold)
    g. Extension of concepts to a Two-body example
3. Bifurcations (1D)
    a. Bifurcation diagrams
    b. Saddle-node, transcritical, and pitchfork bifurcations
4. Linear Systems (2D)
    a. Phase portraits
    b. Stable and unstable manifolds
    c. Classifications of fixed points, applicability to higher order systems
    d. Lyapunov stability (proof)
    e. Properties of eigendirections
    f. Classification of fixed points
5. Phase Planes (nonlinear, 2D and in the CRTBP)
    a. Phase portraits in nonlinear systems
    b. Existence and uniqueness proofs
    c. Comparison with planar CRTBP
    d. Fixed points and linearization
    e. Lagrange points
    f. Symmetry
    g. Basic Periodic orbits
6. Bifurcations (2D and greater)
    a. Hopf bifurcations
    b. Applications within the CRTBP
7. Additional Dynamical Systems Applications in the CRTBP
    a. Periodic and Quasi-Periodic Orbits
        i. Lyapunov Orbits
        ii. Halo Orbits
        iii. Lissajous Orbits
    b. Variational Equations
    c. Single Shooting
    d. Multiple Shooting
        i. Level I
        ii. Level II
        iii. Constraints
    e. Stability Techniques
    f. Invariant Manifolds
    g. Poincaré Maps
8. Applications Within Additional Models (if time allows)
    a. N-Body Problem
    b. Three-Body Problem
    c. Circular Restricted Three-Body Problem
    d. Hill's Problem
    e. Elliptic Restricted Three-Body Problem

    f. Ephemeris Models
9. Other Dynamical Systems Tools in the CRTBP (if time allows)
    a. Heteroclinic and Homoclinic Connections
    b. Regularization
    c. Continuation
10. Examples of Possible Mission Design Applications (if time allows)
    a. Libration Point Missions
    b. Genesis
    c. Earth-Moon System
    d. Comets in the Sun-Jupiter System
    e. Jovian System

Suggested Texts (Ordered roughly by how much they will be used in the course):

[1] Strogatz, S. H., *Nonlinear Dynamics and Chaos*, Perseus Books Publishing, L.L.C., 1994.

[2] Nayfeh, A. H. and B. Balachandran, *Applied Nonlinear Dynamics: Analytical, Computational, and Experimental Methods*, Wiley Series in Nonlinear Science, John Wiley & Sons, Inc., 1995.

[3] Szebehely, V., *Theory of Orbits: The Restricted Problem of Three Bodies*, Academic Press, New York, 1967.

[4] Murray, C. D. and S. F. Dermott, *Solar System Dynamics*, Cambridge University Press, Cambridge, United Kingdom, 1999.

[5] Parker, T. and L. O. Chua, *Practical Numerical Algorithms for Chaotic Systems*, Springer-Verlag, New York, 1989.

[6] Roy, A. E., *Orbital Motion*, 3rd Edition, Adam Hilger, Philadelphia, Pennsylvania, 1988.

[7] Gómez, G., J. Llibre, M. R., and C. Simó, *Dynamics and Mission Design Near Libration Points Vol. I Fundamentals: The Case of Collinear Libration Points*, Volume 2 of *World Scientific Monograph Series in Mathematics*, World Scientific, New Jersey, 2001.

[8] Arnold, V. I., *Mathematical Methods of Classical Mechanics*, Volume 60 of *Graduate Texts in Mathematics*, 2nd Edition, Springer-Verlag, New York, 1989.

[9] Meyer, K. R. and G. R. Hall, *Introduction to Hamiltonian Dynamical Systems and the N-Body Problem*, Springer-Verlag, New York, 1992.

[10] Bate, R. R., D. D. Mueller, and J. E. White, *Fundamentals of Astrodynamics*, Dover Publications, Inc., New York, 1971.

[11] Danby, J. M. A., *Fundamentals of Celestial Mechanics*, 2nd Edition, Willmann-Bell, Inc., 1992.

[12] Landau, L. D. and E. M. Lifshitz, *Mechanics: Course of Theoretical Physics*, Volume 1, 3rd Edition, Pergamon Press, New York, 1976.

[13] Marsden, J. E., *Elementary Classical Analysis*, W. H. Freeman and Company, San Francisco, 1974.

[14] Moulton, F. R., *An Introduction to Celestial Mechanics*, 2nd Edition, The MacMillan Company, New York, 1914.

[15] Pollard, H., *Mathematical Introduction to Celestial Mechanics*, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1966.

[16] Vallado, D. A., *Fundamentals of Astrodynamics and Applications*, Space Technology Series, The MacGraw-Hill Companies, Inc., New York, 1997.

[17] Wiggins, S., *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, Volume 2 of *Texts in Applied Mathematics*, 2nd Edition, Springer-Verlag, New York, 2003.
[18] Wintner, A., *The Analytical Foundations of Celestial Mechanics*, Princeton University Press, Princeton, New Jersey, 1941.

Suggested Papers and References (For research projects):

[1] Bell, J. L., M. W. Lo, and R. S. Wilson, "Genesis Trajectory Design," *AAS/AIAA Astrodynamics Specialist Conference*, Paper AAS 99-398, Girdwood, Alaska, August 16-19, 1999.
[2] Birkhoff, G. D., "The Restricted Problem of Three Bodies," *Rendicoti del Circolo Matematico di Palermo*, Volume 39, pp. 265–334, 1915.
[3] Bray, T. A. and C. L. Goudas, "Doubly Symmetric Orbits about the Collinear Lagrangian Points," *The Astronomical Journal*, Volume 72, Number 2, pp. 202–213, March, 1967.
[4] Broucke, R. A., "Periodic Orbits in the Restricted Three-Body Problem With Earth-Moon Masses," Technical Report 32-1168, Jet Propulsion Laboratory, February 15, 1968.
[5] Conley, C., "On Some New Long Periodic Solutions of the Plane Restricted Three Body Problem," *Comm. Pure Applied Math.*, Volume 16, pp. 449–467, 1963.
[6] Conley, C., "Low Energy Transit Orbits in the Restricted Three-Body Problem," *SIAM Journal of Applied Mathematics*, Volume 16, pp. 732–746, 1968.
[7] Conley, C. C., "The Retrograde Circular Solutions of the Restricted Three-Body Problem Via a Submanifold Convex to the Flow," *SIAM Journal of Applied Math*, Volume 16, Number 3, pp. 620–625, May, 1968.
[8] Conley, C. and R. Easton, "Isolated Invariant Sets and Isolating Blocks," *Transactions of the American Mathematical Society*, Volume 158, Number 1, pp. 35–61, July, 1971.
[9] Dellnitz, M. and O. Junge, "On the Approximation of Complicated Dynamical Behavior," *SIAM Journal on Numerical Analysis*, Volume 36, Number 2, pp. 491–515, 1999.
[10] Dellnitz, M., G. Froyland, and O. Junge, "The algorithms behind GAIO – Set oriented numerical methods for dynamical systems," *Ergodic Theory, Analysis, and Efficient Simulation of Dynamical Systems*, Edited by B. Fiedler, Springer, pp. 145–174, 2001.
[11] Dellnitz, M., O. Junge, M. Lo, and B. Thiere, "On the Detection of Energetically Efficient Trajectories for Spacecraft," *AAS/AIAA Astrodynamics Specialist Conference*, Paper AAS 01-326, Quebec City, Quebec, July 30 - August 2, 2001.
[12] Duncan, M. J., H. F. Levison, and M. H. Lee, "A Multiple Time Step Symplectic Algorithm for Integrating Close Encounters," *The Astronomical Journal*, Volume 116, pp. 2067–2077, October, 1998.
[13] Easton, R. W., "Regularization of Vector Fields by Surgery," *Journal of Differential Equations*, Volume 10, pp. 92–99, 1971.
[14] Easton, R. W., "The Topology of the Regularized Integral Surfaces of the 3-Body Problem," *Journal of Differential Equations*, Volume 12, pp. 361–384, 1972.
[15] Easton, R. W. and R. McGehee, "Homoclinic Phenomena for Orbits Doubly Asymptotic to an Invariant Three-Sphere," *Indiana University Mathematics Journal*, Volume 28, Number 2, pp. 211–240, 1979.
[16] Easton, R. W., *Geometric Methods for Discrete Dynamical Systems*, Oxford University Press, New York, 1998.

[17] Farquhar, R. W., "Station-Keeping in the Vicinity of Collinear Libration Points with an Application to a Lunar Communications Problem," *AAS Science and Technology Series: Space Flight Mechanics Specialist Symposium*, Volume 11, Denver, Colorado, pp. 519–535, July, 1966.

[18] Farquhar, R. W., "Lunar Communications with Libration-Point Satellites," *Journal of Spacecraft and Rockets*, Volume 4, pp. 1383–1384, October, 1967.

[19] Farquhar, R. W., "The Control and Use of Libration-Point Satellites," Technical Report R-346, NASA, September, 1970.

[20] Farquhar, R. W. and A. A. Kamel, "Quasi-Periodic Orbits About the Translunar Libration Point," *Celestial Mechanics*, Volume 7, pp. 458–473, 1973.

[21] Farquhar, R. W., D. P. Muhonen, C. R. Newman, and H. S. Heuberger, "Trajectories and Orbital Maneuvers for the First Libration-Point Satellite," *Journal of Guidance and Control*, Volume 3, Number 6, pp. 549–554, November-December, 1980.

[22] Forest, E. and R. D. Ruth, "Fourth-Order Symplectic Integration," *Physica D Nonlinear Phenomena*, Volume 43, pp. 105–117, May, 1990.

[23] Gómez, G., J. Masdemont, and C. Simó, "Quasihalo Orbits Associated with Libration Points," *Journal of the Astronautical Sciences*, Volume 46, Number 2, pp. 135–176, April-June, 1998.

[24] Gómez, G., A. Jorba, J. Masdemont, and C. Simó, "Study of the Transfer Between Halo Orbits," *Acta Astronautica*, Volume 43, pp. 493–520, 1998.

[25] Hénon, M., "Numerical Exploration of the Restricted Problem, V," *Astronomy and Astrophysics*, Volume 1, pp. 223–238, February, 1969.

[26] Howell, K. C. and J. V. Breakwell, "Three-Dimensional, Periodic, 'Halo' Orbits," *Celestial Mechanics*, Volume 32, 1984.

[27] Howell, K. C. and H. J. Pernicka, "Numerical Determination of Lissajous Trajectories in the Restricted Three-Body Problem," *Celestal Mechanics*, Volume 41, 1988.

[28] Howell, K. C. and H. J. Pernicka, "Stationkeeping Method for Libration Point Trajectories," *Journal of Guidance, Control, and Dynamics*, Volume 16, Number 1, pp. 151–159, January-February, 1993.

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[34] McGehee, R. P., *Some Homoclinic Orbits for the Restricted Three Body Problem*, Ph.D. Thesis, University of Wisconsin, Madison, Wisconsin, 1969.

[35] McGehee, R., "Triple Collision in the Collinear Three-Body Problem," *Inventiones Mathematicae*, Volume 27, pp. 191–227, 1974.

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Conference, Paper AAS 01-303, Quebec City, Quebec, July 30 - August 2, 2001.

[37] Pernicka, H. J., *The Numerical Determination of Lissajous Orbits in the Circular Restricted Three-Body Problem*, Master's Thesis, Purdue University, December, 1986.

[38] Pernicka, H. J., *The Numerical Determination of Nominal Libration Point Trajectories and Development of a Station-Keeping Strategy*, Ph.D. Thesis, 1990.

[39] Pernicka, H. J. and K. C. Howell, "Sun-Earth Libration Point Trajectories that Avoid the Solar Exclusion Zone," *Journal of the Astronautical Sciences*, Volume 38, Number 3, pp. 269–288, 1990.

[40] Richardson, D. L. and N. D. Cary, "A Uniformly Valid Solution for Motion About the Interior Libration Point of the Perturbed Elliptic-Restricted Problem," *Paper AAS 75-021, AAS/AIAA Astrodynamics Specialist Conference*, Nassau, Bahamas, July 28-30, 1975.

[41] Richardson, D. L., "Analytic Construction of Periodic Orbits About the Collinear Points," *Celestial Mechanics*, Volume 22, pp. 241–253, 1980.

[42] Richardson, D. L., "A Note on a Lagrangian Formulation for Motion About the Collinear Points," *Celestial Mechanics*, Volume 22, pp. 231–236, 1980.

[43] Ruth, R. D., "A Canonical Integration Technique," *IEEE Transactions on Nuclear Science*, Volume 30, pp. 2669–2671, August, 1983.

[44] Simó, C., G. Gómez, J. Llibre, and R. Martínez, "Stationkeeping of a Quasiperiodic Halo Orbit Using Invariant Manifolds," *Second International Symposium on Spacecraft Flight Dynamics*, European Space Agency, Darmstadt, Germany, pp. 65–70, October, 1986.

[45] Simó, C., G. Gómez, J. Llibre, R. Martínez, and R. Rodríquez, "On the Optimal Station Keeping Control of Halo Orbits," *Acta Astronautica*, Volume 15, Number 6, pp. 391–397, 1987.

[46] Sweetser, T. H., "Estimate of the Global Minimum DV Needed for Earth-Moon Transfer," *AIAA/AAS Spaceflight Mechanics Meeting*, 91-101, Houston, TX, February, 1991.

[47] Szebehely, V., "Review of the Dynamical Aspects of Triple Systems," *Rev. Mex. Astron. Astrofis.*, Volume 3, May, 1977.

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